Unconventional Monetary Policy and Financial Stability

A Dynamic Stochastic General Equillibrium (DSGE) Approach

Macroeconomics 871

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Due: 20/05/2019

Abstract

This paper develops a quantitative monetary DSGE model with financial intermediaries that face endogenously determined balance sheet constraints. From this, the framework is used to assess the extent in which unconventional monetary policy measures carried out by the Fed have been able to mitigate the disruptive effects which arise during a financial crisis. Within the model, unconventional monetary policy is interpreted as the expansion of central bank credit intermediation to offset a disruption of private financial intermediation. The results indicate that credit easing can be a highly beneficial policy response during a financial crisis, even in the event that the nominal interest rate has not reached the zero lower bound. However, in the event in which the zero lower bound constraint is binding, the welfare benefits from this policy measure are enhanced substantially.

1 Introduction

In 2007, the sub-prime mortgage crisis in the US triggered the largest global economic freefall experienced since the Great Depression. In the aftermath of a crisis, central bankers have traditionally responded to these pressures through lowering their policy rates as a way of stimulating aggregate demand. However, the post-crisis era has posed a new challenge, where policy rates in advanced economies (AEs) have approached the zero lower bound. In essence, this phenomenon constrains any efforts to improve liquidity purely from lowering interest rates further.

With interest rates constrained at the zero lower bound, AE policymakers have switched their focus towards deploying unconventional measures as a mechanism for combating the crunch in liquidity. These unconventional schemes are collectively known as Quantitative Easing (QE), where central banks in these economies have focused on injecting liquidity into financial markets through large scale asset purchase programs. Even though the implementation of unconventional policy has generated an interest in empirical research, the policy effects on the real economy remain open ended. Putting this into perspective, how should we relate monetary policy indicators to the exogenous components of unconventional monetary policy?

Considering the above, the primary aim of this paper is to develop a quantitative model which evaluates the stochastic dynamics of unconventional monetary policy measures employed by the Fed during the post-Lehman era. The model captures these aspects through the assumption that the Fed's role within the financial system is to act as an intermediary, where it borrows funds from households and loans these funds to non-financial firms. Stochastic dynamics on the quality of capital stocks are evaluated as a proxy through three separate shocks; 1) Total Factor Productivity (TFP), 2) short term interest rates and 3) net worth (which is proxied by the risk premium). This paper finds that through their large scale asset purchase program (a broad part of Quantitative Easing), the Fed has been able to limit the downside effects which the sub-prime mortgage crash has had on TFP, short term interest rates and the risk premium in the US.

The rest of this paper is structured as follows. Section 2 adopts a similar approach as Smets and Wouters (2007) by unpacking the structure of the model. Thereafter, the calibration and results are discussed in Section 3, before offering concluding remarks in Section 4.

2 The Model

The model built for this paper follows a similar framework to seminal studies such as Christiano, Eichenbaum and Evans (2005), Smets and Wouters (2007), and Gertler and Karadi (2011). There is an agency problem which restricts the ability for financial intermediaries to receive finances from households. The model also allows for a disturbance to the quality of assets. In Section 3, this model shows that there is a potential for the central bank to intervene in the financial sector through credit easing.

There are 5 different agents in the model; households, financial intermediaries (the banking sector in this paper's case), non-financial goods producers, capital producers and monopolistically competitive retailers. Finally, there is a central bank which implements both conventional policy measures as well as the unconventional policy measures explained earlier. Where this model differs from Christiano *et al.* (2005) is the inclusion of financial intermediation between households and non-financial firms. Gertler and Karadi (2011) rationale behind the inclusion of financial intermediaries in a model within this context is based on the considerable impacts it has on unconventional monetary policy.

2.1 Households

Within the model, households consume, save and supply labour. Savings are captured by households lending finances to both intermediaries and the government. In unpacking the role of households, they constitute of workers who supply labour (therefore transferring the wages they earn back to their households), and financial managers, who manage financial intermediaries and transfer earnings back to their households. By implication, households own the intermediaries which they manage, but their deposits are in intermediaries which they do not manage. Finally, each household is perfectly insured against idiosyncratic risks to their wealth.

Out of all household members, f of them are financial managers (meaning that $1 - f$ are workers). Individuals can switch between either of these roles over time. The probability of a financial manager staying in his role for the next period is θ, which is unrelated to how long a individual has been a manager for. Therefore, the average lifespan of a manager in any period is $\frac{1}{1-\theta}$. In a given period, $(1-\theta)f$ of managers will become workers in the next period, meaning they would have to transfer their earnings back to their households. Similarly, for every manager that becomes a worker, will result in a worker moving into a financial managerial role. Every new financial manager receives start up finance from their respective household.

Let C_t be consumption and L_t household labour supply. The households preferences are given by

$$\max E_t \sum_{i=0}^{\infty} \beta^i \left[ln(C_{t+i} - hC_{t+i-1}) - \frac{\chi}{1+\varphi} L_{t+i}^{1+\varphi} \right] \tag{1}$$

with $0 < \beta < 1$, $0 < h < 1$ and ξ, $\varphi > 0$. The intermediary deposits and government debt are one period real bonds which pay a gross real return of R_t. It is assumed that these instruments are risk free, and by implication perfect substitutes of each other. If B_{t+1} is the total amount of short term debt for households, W_t is the real wage, Π_t reflects the profits households earn and T_t is a lump sum tax, the household budget constraint is given by

$$C_t = W_t L_t + \Pi_t + T_t + R_t B_t - B_{t+1} \tag{2}$$

Let ϱ_t be the marginal utility of consumption and $\Lambda_{t,t+1}$ be the stochastic discount rate of consumption between period t and $t-1$. As proven in appendix C.1, the households' first order conditions for labour supply and consumption/savings are

$$\chi L_t^{\varphi} = W_t \varrho_t \tag{3}$$

$$1 = \beta E_t (R_{t+1} \Lambda_{t,t+1}) \tag{4}$$

2.2 Financial Intermediaries

Financial intermediaries use the deposits they receive from households and lend these to non financial firms. The intermediaries keep long-term assets on their balance sheets, which they fund using short-term liabilities. For understanding purposes, financial intermediaries in this model comprise of the whole banking sector.

If $N_{j,t}$ is the net worth of a specific intermediary for a specific period, $B_{j,t+1}$ denotes the amount of liabilities an intermediary acquires from households, $S_{j,t}$ is an intermediary's amount of financial claims on non financial assets, with Q_t being the price of these claims, then the intermediary balance sheet is given by

$$Q_t S_{j,t} = N_{j,t} + B_{j,t+1} \tag{5}$$

As highlighted previously, the return on household deposits is denoted by R_{t+1}. Therefore, $B_{j,t+1}$ relates to an intermediary's debt with $N_{j,t}$ representing it's equity. An intermediary's return on assets can be denoted as $R_{k,t+1}$. Each intermediary's equity is calculated as the difference between the return on its assets and the interest payments on debt

$$N_{j,t+1} = R_{k,t+1} Q_t S_{j,t} - R_{t+1} B_{j,t+1} \tag{6}$$

$$= (R_{k,t+1} - R_{t+1}) Q_t S_{j,t} + R_{t+1} N_{j,t} \tag{7}$$

$$E_t \beta^i \Lambda_{t,t+1+i} (R_{k,t+1+i} - R_{t+1+i}) \geq 0, \qquad i \geq 0$$

The growth in equity above the risk free return depends on the financial managers premium earned on its assets, $R_{k,t+1} - R_{t+1}$, and the total quantity assets, $Q_t S_{j,t}$. If $\beta^i \Lambda_{t,t+i}$ is the discount applied to the financial managers earnings, the following inequality applies:

$$E_t \beta^i \Lambda_{t,t+1+i} (R_{k,t+1+i} - R_{t+1+i}) \geq 0, \qquad i \geq 0$$

The financial manager's objective to maximise its expected terminal wealth is given by

$$\begin{aligned} V_{j,t} &= maxE_t \sum_{i=0}^{\infty}(1-\theta)\theta^i\beta^{i+1}\Lambda_{t,t+1+i}(N_{j,t+1+i}) \\ &= maxE_t \sum_{i=0}^{\infty}(1-\theta)\theta^i\beta^{i+1}\Lambda_{t,t+1+i}\left[(R_{k,t+1+i}-R_{t+1+i})Q_{t+1}S_{j,t+1}+R_{t+1+i}N_{j,t+1}\right] \end{aligned} \tag{8}$$

Depending on how large the discounted risk adjustment premium $(\beta^i\Lambda_{t,t+1+i}(R_{k,t+1+i} - R_{t+1+i}))$ is, the intermediary will aim to increase the assets they hold by borrowing further from households. However, there needs to be a limit as to how much the intermediary can undertake in additional debt. To incorporate this constraint, a moral hazard problem is introduced whereby the financial manager can choose to redirect λ of available funds from an investment and transfer it back to their household. With this, the financial manager's cost would be that the depositors can force the intermediary into bankruptcy and recover the remainder of the assets, $1-\lambda$. For depositors, recovering λ would be too expensive.

In order for these depositors to be willing to supply funds to financial managers, the following constraint must apply

$$V_{j,t} \geq \lambda Q_t S_{j,t} \tag{9}$$

The left hand side of the above inequality denotes the financial manager's cost if they redirected a portion of assets. The right hand side shows what they would gain from doing so. $V_{j,t}$ can be expressed as follows

$$V_{j,t} = \nu \cdot Q_t S_{j,t} + \eta_t N_{j,t} \tag{10}$$

with

$$\begin{aligned} \nu_t &= E_t\left[(1-\theta)\beta\Lambda_{t,t+1}(R_{k,t+1}-R_{t+1}) + \beta\Lambda_{t,t+1}\theta x_{t,t+1}\nu_{t+1}\right] \\ \eta_t &= E_t\left[(1-\theta) + \beta\Lambda_{t,t+1}\theta z_{t,t+1}\eta_{t+1}\right] \end{aligned} \tag{11}$$

where $x_{t,t+i} \equiv {}^{Q_{t+i}S_{j,t+i}}/_{Q_t S_{j,t}}$ refers to the gross growth in assets between t and $t+i$, and $z_{t,t+i} \equiv {}^{N_{j,t+i}}/_{N_{j,t}}$ is the gross growth rate of net worth. ν_t is the expected discounted marginal benefit to the financial manager for growing their assets by 1 unit, assuming net worth remains constant. Similarly, η_t is the expected discounted value of holding another unit of $N_{j,t}$, assuming $S_{j,t}$ remains constant. Below, it will be shown that in the case of binding incentive constraints, the intermediary's assets are constrained by its equity capital. The incentive constraint is as follows

$$\eta_t N_{j,t} + \nu_t Q_t S_{j,t} \geq \lambda Q_t S_{j,t} \tag{12}$$

Assuming this constraint is binding, the assets which a financial manager can acquire is contingent on their equity can be represented as

$$\begin{aligned} Q_t S_{j,t} &= \frac{\eta_t}{\lambda - \nu_t} N_{j,t} \\ &= \phi_t N_{j,t} \end{aligned} \tag{13}$$

where ϕ_t is the private leverage ratio. If $N_{j,t}$ is constant, expanding $S_{j,t}$ increases the financial managers' incentive to redirect funds. The constraint limits the leverage ratio to the point where the financial managers' incentive to deviate is equal to the cost thereof. By implication, the agency problem results in an endogenously determined capital constraint for the intermediary's balance sheet.

If $N_{j,t} > 0$, the constraint is only binding where $0 < \nu_t < \lambda$. Should this be the case, it is profitable for the financial manager to acquire more assets (since $\nu_t > 0$). The larger ν_t, the larger the opportunity cost for the financial manager to be forced into bankruptcy. If $\nu_t > \lambda$, the constraint will be non-binding. In this model, the constraint always binds around the steady state. The financial manager's net worth evolves as follows

$$N_{j,t+1} = \left[(R_{k,t+1} - R_{t+1})\phi_t + R_{t+1}\right] N_{j,t} \tag{14}$$

Furthermore, it follows that

$$\begin{aligned} z_{t,t+1} &= N_{j,t+1}/N_{j,t} \\ &= (R_{k,t+1} - R_{t+1})\phi_t + R_{t+1} \end{aligned}$$

$$\begin{aligned} x_{t,t+1} &= Q_{t+1} S_{j,t+2}/Q_t S_{t+1} \\ &= (\phi_{t+1}/\phi_t)(N_{j,t+1}/N_t) \\ &= (\phi_{t+1}/\phi_t) z_{t,t+1} \end{aligned}$$

From this, it can be seen that all of the components of ϕ_t are independent of firm specific factors. By summing the demand of assets for individual intermediaries, the following relation holds

$$Q_t S_t = \phi_t N_t \tag{15}$$

where S_t denotes the aggregate quantity of intermediary assets and N_t denotes aggregate intermediary capital. In general equilibrium, fluctuations in N_t will lead to fluctuations in overall asset demand by intermediaries. A crisis will lead to a rapid decrease in N_t. The behaviour of N_t over time is determined as follows

$$N_t = N_{e,t} + N_{n,t} \tag{16}$$

where $N_{e,t}$ is this net worth of existing intermediaries, and $N_{n,t}$ is the net worth of new financial managers in a specific period. Since θ of financial managers at $t-1$ remain in their positions until period t, $N_{e,t}$ follows

$$N_{e,t} = \theta\left[(R_{k,t} - R_t)\phi_{t-1} + R_t\right] N_{t-1} \tag{17}$$

Extending on equation 17, most of the variation in $N_{e,t}$ will be from the variance in the ex post return on assets, $R_{k,t}$. Further, the impact on $N_{e,t}$ increases with the leverage ratio.

As highlighted previously, new financial managers obtain seed funding from their households. This seed funding is equal to a small portion of the value of assets that the exiting financial managers had intermediated in their final period. The final period assets of financial managers which exit is denoted as $(1-\theta)Q_tS_{t-1}$. Thus, the net worth of new financial managers is as follows

$$N_{n,t} = \omega Q_t S_{t-1} \tag{18}$$

The combination of equations 17 and 18 leads to the following equation of motion for N_t

$$N_t = \theta\left[(R_{k,t} - R_t)\phi_{t-1} + R_t\right] N_{t-1} + \omega Q_t S_{t-1}$$

This shows that ω drives down the steady state leverage ratio QS/N. In Section 3, ω is calibrated to further illustrate this point.

2.3 Credit Policy

In this section, it is assumed that the central bank is willing to facilitate lending. Let $Q_tS_{g,t}$ denote the value of public intermediated assets, and Q_tS_t is the total value of intermediated assets, then

$$Q_tS_t = Q_tS_{p,t} + Q_tS_{g,t} \tag{19}$$

The central bank issues public debt to households that pays the risk free rate R_{t+1}, and then extends these funds to non-financial firms at the market determined interest rate $R_{k,t+1}$. The model assumes that this central bank credit includes an efficiency cost of τ per unit. This cost could relate to the cost of raising capital through public debt, or the cost the central bank faces to identify investments in the private sector. Contrasting to the private intermediation case, public intermediation is not constrained by its balance sheet. This is because the public sector always fulfils its debt obligations, meaning that they don't face an agency problem compared to the private intermediation case.

Instead of issuing debt to households first, the central bank can also extend government debt to financial intermediaries. Intermediaries would then fund their government liabilities by issuing deposits to households. Assuming the agency problem is only applicable to private assets, this would mean that a financial intermediary faces no constraints on funding its government debt.

Assuming that the central bank is willing to fund the fraction ψ_t of intermediated assets, then

$$Q_t S_{g,t} = \psi_t Q_t S_t \tag{20}$$

The central bank issues government bonds $B_{g,t}$ equal to the right hand side of equation 19 in order to fund this. The net earnings from intermediation is therefore equal to $(R_{k,t+1} - R_{t+1})B_{g,t}$. As highlighted previously, privately intermediated funds are constrained by intermediary net worth, so that equation 19 can be rewritten as

$$\begin{aligned} Q_t S_t &= \phi_t N_t + \psi_t Q_t S_t \\ &= \phi_{c,t} N_t \end{aligned}$$

where ϕ_t denotes the leverage ratio for privately intermediated funds (as in equation 13). Additionally, $\phi_{c,t}$ is the leverage ratio for total intermediated funds, given as:

$$\phi_{c,t} = \frac{1}{1 - \psi_t} \phi_t$$

2.4 Intermediate Goods Firms

This section analyses the production and investment side of the economy. Competitive non-financial firms produce intermediate goods which are then sold to retail firms. At the end of a specific period t, an intermediate goods firm acquires capital K_{t+1} to use in the following period. After they produce in period $t+1$, they can then sell their capital. Finally, the firms do not face any adjustment costs.

The firm funds its capital requirement in each period through obtaining funds from intermediaries. In order to do this, the firm needs to issue S_t claims equal to the amount of capital it acquires, and prices each claim at Q_t. By implication, $Q_t K_{t+1}$ denotes the value of capital which it has acquired and $Q_t S_t$ is the value of claims against the capital in question. Therefore,

$$Q_t K_{t+1} = Q_t S_t \tag{21}$$

The model suggests that there are no frictions from non-financial firms obtaining funds from intermediaries. The intermediary has perfect information about this firm, which is different to when the intermediary obtains funds from households (in this case, there are financial frictions). The main implication for this is that only intermediaries are faced with capital constraints when obtaining funds. These constraints affect the availability of funds to non financial firms and the return on capital which the above mentioned firms are required to pay.

For each specific period, the firm produces output (denoted by Y_t) using capital and labour inputs (K_t and L_t), and by varying the utilisation rate of capital (U_{t+1}). If A_t refers to total factor productivity and ξ_t is the quality of capital, so that $\xi_t K_t$ is the effective quantity of capital in period t, then the firms production is determined as

$$Y_t = A_t (U_t \xi_t K_t)^\alpha L_t^{1-\alpha} \tag{22}$$

The shock ξ_t depicts the exogenous variation in the value of capital, which arises from depreciation. However, this model specifies that the market value of capital is determined endogenously. If $P_{m,t}$ is the price of intermediate goods output, and the replacement price of used capital is fixed at 1, then the firm's utilisation rate and demand for labour are determined as follows

$$P_{m,t} \alpha \frac{Y_t}{U_t} = \delta'(U_t) \xi_t K_t \tag{23}$$

$$P_{m,t} (1-\alpha) \frac{Y_t}{L_t} = W_t \tag{24}$$

Due to the fact that the firm earns zero profit state by state, it merely pays out the ex post return to capital to the intermediary. This return, $R_{k,t+1}$, is given by

$$R_{k,t+1} = \frac{\left[P_{m,t+1} \alpha \frac{Y_{t+1}}{\xi_{t+1} K_{t+1}} + Q_{t+1} - \delta(U_{t+1}) \right] \xi_{t+1}}{Q_t} \tag{25}$$

Due to the fact that the replacement price of capital that has depreciated is 1, the value of the remaining capital stock is $(Q_{t+1} - \delta(U_{t+1}))\xi_{t+1}K_{t+1}$. This shows that the valuation shock explains the variation in the return on capital. Additionally, present asset prices will be dependant on the expectations of ξ_{t+i}.

2.5 Capital Producing Firms

At the end of a specific period t, competitive capital firms produce new capital stocks through buying capital from intermediate goods firms and refurbishing it for resale, as well as building new capital. Assuming that the value of a new unit of capital is Q_t, and adjustment costs only apply to the production of new capital and not the refurbishment of existing capital stocks. Let I_t be the gross capital created, $I_{n,t}$ denotes the amount of net capital, and I_{ss} is investment in the steady state, then the discounted profits for a capital producer would be

$$max E_t \sum_{\tau=t}^{\infty} \beta^{T-\tau}\Lambda_{t,\tau}\left[(Q_t - 1)I_{n,\tau} - f\Big(\frac{I_{n,\tau}+I_{ss}}{I_{n,\tau-1}+I_{ss}}\Big)(I_{n,\tau}+I_{ss})\right] \tag{26}$$

with

$$I_{n,t} \equiv I_t - \delta(U_t)\xi_t K_t$$

where $f(1) = f'(1) = 0$ and $f''(1) > 0$, and $\delta(U_t)\xi_t K_t$ denotes the quantity of refurbished capital. Similar to Christiano *et al.* (2005), there are adjustment costs for the flow of investment, however this cost only applies to net investment flows. Therefore, capital producers can earn profits outside of the steady state. In the end, these profits are then redirected back to households. It is important to account for the fact that these capital producing firms choose the same net investment rate.

As proved in Appendix C.3, the first order condition for investment is given by the following equation:

$$Q_t = 1 + f(\cdot) + \frac{I_{n,\tau}+I_{ss}}{I_{n,\tau-1}+I_{ss}} f'(\cdot) - E_t\beta\Lambda_{t,t+1}\Big(\frac{I_{n,t+1}+I_{ss}}{I_{n,t}+I_{ss}}\Big)^2 f'(\cdot) \tag{27}$$

2.6 Retail Firms

Total final output, denoted by Y_t, is a Constant Elasticity of Substitution (CES) composite given by the following

$$Y_t = \left[\int_0^1 Y_{f,t}^{(\varepsilon-1)/\varepsilon} df\right]^{\varepsilon/(\varepsilon-1)} \tag{28}$$

where $Y_{f,t}$ refers to the output for a specific retail firm f. From cost minimisation by users of final output, the following relations hold:

$$Y_{f,t} = \left(\frac{P_{f,t}}{P_t}\right)^{-\varepsilon} Y_t \tag{29}$$

$$P_t = \left[\int_0^1 P_{f,t}^{(1-\varepsilon)} df\right]^{1/(1-\varepsilon)} \tag{30}$$

One unit of intermediate output is needed to produce a unit of retail output, therefore retailers only repackage the output from the intermediate production phase. The marginal cost for retailers is simply the relative price of intermediate goods, $P_{m,t}$. In each period, a firm can adjust its price with the probability $(1-\gamma)$. The firms prices are determined by the lagged rate of inflation. The retailers solve its optimal pricing problem by the following equation:

$$\max E_T \sum_{i=0}^{\infty} \gamma^i \beta^i \Lambda_{t,t+i} \left[\frac{P_t^*}{P_{t+i}} \prod_{k=1}^{i} (1+\pi_{t+k-1})^{\gamma_p} - P_{m,t+i}\right] Y_{f,t+i} \tag{31}$$

where π_t is the inflation rate from $t-i$ to t. The first order condition is given as follows

$$\sum_{i=0}^{\infty} \gamma^i \beta^i \Lambda_{t,t+i} \left[\frac{P_t^*}{P_{t+i}} \prod_{k=1}^{i} (1+\pi_{t+k-1})^{\gamma_p} - \mu P_{m,t+i}\right] Y_{f,t+i} = 0 \tag{32}$$

with

$$\mu = \frac{1}{1-1/\varepsilon}$$

From the law of large numbers, the following relation for the evolution of the price level applies

$$P_t = \left[(1-\gamma)(P_t^*)^{1-\varepsilon} + \gamma(\pi_{t-1}^{\gamma_p} P_{t-1})^{1-\varepsilon}\right]^{1/(1-\varepsilon)} \tag{33}$$

2.7 Resource Constraint and Government Policy

Y_t is divided between consumption, investment, government consumption, G_t, and expenditure related to government intermediation, denoted by $\tau\psi Q_t K_{t+1}$. It is assumed that government expenditure is exogenously fixed at G. The economy-wide resource constraint follows

$$Y_t = C_t + I_t + f\left(\frac{I_{n,t} + I_{ss}}{I_{n,t-1} + I_{ss}}\right)(I_{n,t} + I_{ss}) + G + \tau\psi_t Q_t K_{t+1} \tag{34}$$

where the evolution of capital follows

$$K_{t+1} = \xi_t K_t + I_{n,t} \tag{35}$$

Government expenditure is funded by the combination of lump sum taxes and government intermediation as given by the following equation

$$G + \tau\psi_t Q_t K_{t+1} = T_t + (R_{k,t} - R_t)B_{g,t-1} \tag{36}$$

where government bonds $B_{g,t-1}$ finance total government intermediated assets, $Q_t\psi_{t-1}S_{t-1}$.

If monetary policy follows the Taylor rule with interest rate smoothing, where i_t denotes the net nominal interest rate, i refers to the steady state nominal rate and Y_t^* is the natural level of output. The net nominal interest rate is determined by

$$i_t = (1-\rho)[i + \kappa_\pi \pi_t + \kappa_y(\log Y_t - \log Y_t^*)] + \rho i_{t-1} + \varepsilon_t \tag{37}$$

where $0 \leq \rho \leq 1$ is a smoothing parameter, and ε_t is an exogenous shock to monetary policy. The seminal Fisher relation, equation 38, is applied to explain the relationship between nominal and real interest rates, as follows

$$1 + i_t = R_{t+1}\frac{E_t P_{t+1}}{P_t} \tag{38}$$

It is assumed that monetary policy should follow the Taylor Rule in normal conditions. However, in a crisis the model allows credit policy to occur. Specifically, in the event of a crisis (when credit spreads rise uncontrollably), the central bank should presumably respond to this by injecting credit into the economy. This action follows this specific rule

$$\psi_t = \psi + \nu E_t\big[(\log R_{k,t+1} - \log R_{t+1}) - (\log R_k - \log R)\big] \tag{39}$$

where ψ is the steady state coefficient on public intermediated assets and $\log R_k - \log R$ is the steady state premium. The feedback parameter is always positive, therefore the central bank increases credit as the credit spread increases relative to its value in the steady state.

Finally, the model assumes that the central bank does not smooth interest rates in the event of a crisis. The implication of this is that the model isolates the central bank's behaviour as the crisis unfolds. Unpacking this idea further, the result of interest smoothing is that monetary policy's impact is contingent on the expected direction for short term interest rates.

The decision to set the smoothing parameter equal to zero is rationalised by the fact that the central bank loses its capability to manage expectations during a crisis, meaning it would adjust the present interest rate more rapidly.

3 Model Analysis

3.1 Calibration

This section presents the parameter estimates used in the model which were calibrated based on figures and comparable studies from the United States. The values used are a combination of conventional values as well as values adapted from the studies of Primiceri, Schaumburg and Tambalotti (2006) and Gertler and Karadi (2011). These can be found in Appendix B.

The household parameters consist of: the discount rate, β, the habit formation parameter, h, the relative utility weight of labour, χ, and the inverse Frisch elasticity of labour supply, φ. The financial intermediary parameters consist of - the proportion of capital that can be diverted, λ, the proportional transfer to entering, banks, ζ, and the survival rate of banks, θ. The intermediate goods firm parameters include: the effective capital share, α, the depreciation rate and the elasticity of marginal depreciation with respect to the utilization rate U. Additionally, the capital utilisation rate is set to unity in the steady state. The capital producing firms parameter is the inverse of net investment to the price of capital. The retail firms parameters include - the elasticity of substitution, ε, the probability of keeping prices fixed, γ, and the measure of price indexation, γ_p. Lastly, the government parameters include: the inflation coefficient of the Taylor Rule, κ_π, the output gap coefficient of the Taylor Rule κ_y, the smoothing parameter of the Taylor Rule ρ_i, and the steady state proportion of government expenditures ${}^{G}/_{Y}$.

Of the above parameters, the discount factor, β, depreciation rate, δ, capital share, α, the elasticity of substitution between goods, ε and the government expenditure share are conventional values. According to Gertler and Karadi (2011), the financial intermediary parameters were chosen in order to achieve the following targets - "a steady state interest rate spread of one hundred basis points; a steady state leverage ratio of four; and an average horizon of bankers of a decade." Their paper highlights that the steady state interest rate spread is based on the spread between mortgage rates and government bonds and between BAA corporate compared to government bonds before 2007. The steady state leverage ratio of 4 was targeted as it is an average of the "assets to equity" over a number of sectors during the crisis.

3.2 Experiments and Results

The model built for this paper attempts to understand the effect that an unanticipated negative shock to asset quality would have on the economy if the central bank is allowed to use an unconventional monetary policy, in the form of large-scale asset purchases. In order to build a proxy for this, the model runs three separate estimations in order to determine how output, investment and the risk premium would behave in response to the three different shocks, including a technology shock, a monetary shock, as well as a shock to the central bank's net worth. Figure 1 shows the impulse response functions to these shocks respectively.

The technology shock was a 1% contraction in Total Factor Productivity (TFP) with a quarterly autoregressive factor of 0.95. A decrease in TFP is expected to result in a fall in aggregate demand. The model's results support this theory. The shock resulted in an immediate 0.5% decline in output which persisted for just under a decade, reaching a minimum of more than a 1% decrease in output at around 2 years following the shock. With regards to investment - it declined by 2% immediately after the technology shock. The persistence in this decline was less severe than the output case, with it converging back to its steady state value after 5 years. Lastly, the shock resulted in an immediate 0.6% increase in the risk premium. Following this, it declined consistently throughout the entire period before converging on its steady state after 10 years.

The next shock illustrates the effects arising from a 0.25% increase in the short-term interest rate. Initially, this shock resulted in an initial 0.3% decline in output. This decline persisted throughout the entire timespan, reaching a maximum drop of 0.55% after one year. Following this minimum, output began to converge to its steady state, reaching it after a decade. This magnitude of the effect on investment was much more intense compared to the output case, with an initial fall of 2% reaching a trough of 3% after about 3 periods. This fall quickly began to reverse, reaching its steady state after approximately 4 years. In explaining the effect the shock had on the risk premium, an initial rise of 0.6% quickly began to decrease before converging on its steady state value after 5 years.

The shock to the central bank's net worth was a 1% fall in intermediaries net worth (which is transferred to households). Initially, output fell by approximately 0.7% as a result of this shock. This decrease persisted for the next few periods, reaching a maximum drop of 1.7% after roughly a year, before experiencing an upswing towards its steady state after 20 quarters. The effect on investment were more severe compared to output (similar to the TFP shock), seeing a 1% decrease after a year. Investment converged towards its steady state value after 5 years.

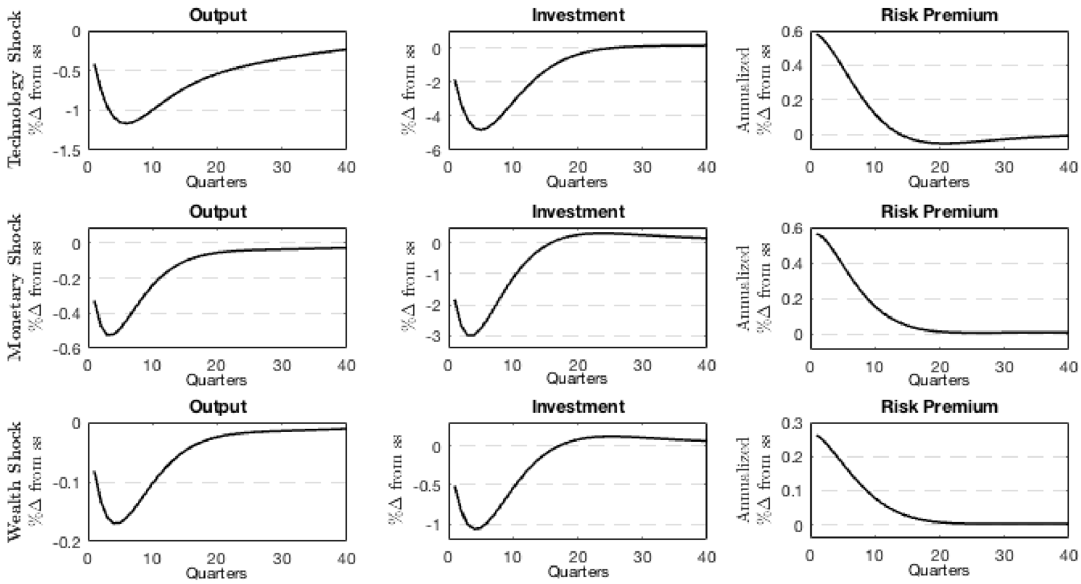

Figure 1: Impulse Response Functions of Output, Investment and the Risk Premium to a negative Technology shock, Monetary shock, and Wealth shock

Lastly, the initial 25% increase in the risk premium was offset after 20 periods, before returning to its steady state.

Figure 1 shows how each of these shocks can be reigned in through the use of unconventional monetary policy. The short persistence of a decrease in each of the impulse response functions is as a result of the model allowing the central bank to utilise unconventional monetary policy, borrowing funds from savers and lending them to investors. While private intermediaries face constraints on their leverage ratios, the central bank does not. Thus, in this model, the central bank has full mobility of borrowing and lending, and halts the negative impact of each of the shocks.

4 Conclusion

The 2007 sub-prime market crash significantly altered the framework of not only the global economy, but individual economies alike. Like many of the big crises that have emerged throughout history, policymakers and economists have been forced to improvise and adapt in order to counter new technologies and agents that may disrupt the stability of the financial system as a whole. During the post-Lehman era, central banks have been forced into using unconventional measures, where the probability of success for these responses has been scrutinized heavily at times.

This paper attempted to replicate the effect that a shock to the quality of assets would have on the financial system, while allowing the Fed to use unconventional monetary policy with full discretion. By relaxing the financial constraints which the Fed are faced with, the model found that shocks to total factor productivity (TFP), short-term interest rates and the risk premium have significant downside effects in its initial stages before experiencing a sharp recovery. This is an encouraging result as economists look to build their arsenal with policies that can counter potential financial destabilising agents.

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A List of Variables

Variable	Description	Variable	Description
C_t	Consumption	L_t	Labour supply
W_t	Real wage	Π_t	Net Payouts
T_t	Lump sum tax	R_t	Gross real return
B_t	Short term debt	ϱ_t	Marginal utility of consumption
Λ	Earnings multiplier	$N_{j,t}$	Net worth of the intermediary
$B_{j,t}$	Intermediary debt	$S_{j,t}$	Number of financial claims on non-financial firms
Q_t	Price of the financial claims	$R_{k,t}$	Return on assets for intermediary
$Q_t S_t$	Total quantity of assets for intermediary	$x_{t,t+i}$	Gross growth rate in assets
$z_{t,t+i}$	Gross growth rate of net worth	ϕ_t	Private leverage ratio
N_t	Aggregate capital of intermediary	$N_{e,t}$	Net worth of existing intermediaries
$N_{n,t}$	Net worth of new intermediaries	${}^{QS}/_{N}$	Steady state leverage ratio
$S_{p,t}$	Quantity of privately intermediated assets	$S_{g,t}$	Quantity of government intermediated assets
τ	Efficiency cost	ψ_t	Coefficient on intermediated assets
$\phi_{c,t}$	Total leverage rate for intermediated assets	K_t	Capital for intermediate goods producer
A_t	Total Factor Productivity (TFP)	$P_{m,t}$	Price of intermediate goods output
Y_t	Output of firm	I_t	Gross capital
I_{ss}	Steady state capital	$I_{n,t}$	Net capital
$Y_{f,t}$	Output of retailer	$P_{f,t}$	Price for retailer goods
π_t	Inflation rate	G_t	Government expenditure
i_t	Nominal Interest rate		

B Table of Parameters

Parameters	Calibrated Value	Description
Households		
β	0.99	Discount Rate
h	0.815	Habit Formation Parameter
χ	3.409	Relative utility weight of labour
φ	0.276	Inverse Frisch elasticity of labour supply
Financial Intermediaries		
λ	0.381	Proportion of capital that can be diverted
ω	0.002	Proportional transfer to entering banks
θ	0.972	Survival rate of banks
Intermediate Good Firms		
α	0.33	Effective capital share
δ	0.025	Depreciation rate
ζ	7.2	Elasticity of marginal depreciation with respect to utilisation rate
Capital Producing Firms		
η_i	1.728	Inverse elasticity of net investment to the price of capital
Retail Firms		
ε	4.167	Elasticity of substitution
γ	0.779	Probability of keeping prices fixed
γ_p	0.241	Measure of price indexation
Government		
κ_π	1.5	Inflation coefficient of the Taylor Rule
κ_y	0.5/4	Output gap coefficient of the Taylor Rule
ρ_i	0.8	Smoothing parameter of the Taylor Rule
${}^G/_Y$	0.2	Steady state proportion of government expenditures

C Technical Appendix

In order to find the first order conditions and linearise them, the preference equations of each of the agents in the model are written as a value function, with the plan of setting up and solving the Bellman's equation.

C.1 Households

The households' preference function is given by

$$\max E_t \sum_{i=0}^{\infty} \beta^i \left[ln(C_{t+i} - hC_{t+i-1}) - \frac{\chi}{1+\varphi} L_{t+i}^{1+\varphi} \right] \tag{1}$$

where

$$C_t = W_t L_t + \Pi_t + T_t + R_t B_t - B_{t+1} \tag{2}$$

The value function for households is therefore

$$V(B_t) = \max_{C_t, L_t, B_{t+1}} \ln(C_t - hC_{t-1}) - \frac{\chi}{1+\varphi} L_{t+i}^{1+\varphi} + \beta E_t V(B_{t+1})$$

s.t.:

$$C_t = W_t L_t + \Pi_t + T_t + R_t B_t - B_{t+1}$$

Let:

$$\varrho_t = \frac{1}{C_t - hC_{t-1}} - h\beta E_t \frac{1}{C_{t+1} - hC_t}$$

and:

$$\Lambda_{t,t+1} \equiv \frac{\varrho_{t+1}}{\varrho_t}$$

C.1.1 Labour Supply

$$\begin{aligned}
\frac{\delta V(B_t)}{\delta L_t} &= 0 \\
0 &= \frac{1}{C_t - hC_{t-1}} W_t - \chi L_t^{\varphi} + \beta E_t \frac{\delta V(B_{t+1})}{\delta L_t} \\
0 &= \frac{1}{C_t - hC_{t-1}} W_t - \chi L_t^{\varphi} + \beta E_t \left[\frac{1}{C_{t+1} - hC_t} (-hW_t) \right] \\
0 &= \frac{1}{C_t - hC_{t-1}} W_t - \chi L_t^{\varphi} + \beta E_t \left[- \frac{1}{C_{t+1} - hC_t} hW_t \right] \\
\chi L_t^{\varphi} &= W_t \left(\frac{1}{C_t - hC_{t-1}} - h\beta E_t \frac{1}{C_{t+1} - hC_t} \right) \\
\chi L_t^{\varphi} &= W_t \varrho_t
\end{aligned} \tag{3}$$

In order to solve for the dynamics of the equation, the components of the labour supply equation are written in their steady state values and then the model is linearised as such:

$$\begin{aligned}
\chi L_{ss}^{\varphi} &= W_{ss} \left(\frac{1}{C_{ss} - hC_{ss}} - h\beta \frac{1}{C_{ss} - hC_{ss}} \right) \\
\chi L_{ss}^{\varphi} &= W_{ss}(1 - h\beta) \left(\frac{1}{C_{ss}} \right) \frac{1}{1-h} \\
\therefore \varphi \hat{l}_{ss} &= \hat{w}_{ss} - \hat{c}_{ss} \\
\therefore \hat{c}_{ss} &= \hat{w}_{ss} - \varphi \hat{l}_{ss}
\end{aligned}$$

C.1.2 Savings

$$\begin{aligned}
\frac{\delta V(B_t)}{\delta B_{t+1}} &= 0 \\
0 =& \frac{1}{C_t - hC_{t-1}}(-1) + \beta E_t \frac{\delta V(B_t + 1)}{\delta B_{t+1}} \\
0 =& - \frac{1}{C_t - hC_{t-1}} + \beta E_t \left[\frac{1}{C_{t+1} - hC_t} (R_{t+1} - h(-1)) + \beta E_{t+1} \left[\frac{1}{C_{t+2} - hC_{t+1}} (-hR_{t+1}) \right] \right] \\
0 =& - \frac{1}{C_t - hC_{t-1}} + \beta E_t \left[\frac{1}{C_{t+1} - hC_t} h \right] + \beta E_t \left[\frac{1}{C_{t+1} - hC_t} R_{t+1} - h\beta E_{t+1} \frac{1}{C_{t+2} - hC_{t+1}} R_{t+1} \right] \\
0 =& - \varrho_t + \beta E_t \left[R_{t+1} \varrho_{t+1} \right]
\end{aligned}$$

$$1 = \beta E_t (R_{t+1} \Lambda_{t,t+1}) \tag{4}$$

In the steady state, this relationship is given by

$$
\begin{aligned}
1 &= \beta R_{ss} \frac{\varrho_{ss}}{\varrho_{ss}} \\
&= \beta R_{ss} \\
\therefore \beta &= \frac{1}{R_{ss}}
\end{aligned}
$$

Thus, the discount rate, β is equal to the inverse of the return on household assets in the steady state.

C.2 Financial Intermediaries

Financial intermediaries seek to maximise their expected terminal wealth. The optimisation relation is therefore a function of the probability of bankers remaining, θ, or exiting, $(1-\theta)$, the industry as given by:

$$
\begin{aligned}
V_{j,t} &= \max E_t \sum_{i=0}^{\infty} (1-\theta)\theta^i \beta^{i+1} \Lambda_{t,t+1+i} (N_{j,t+1+i}) \\
&= \max E_t \sum_{i=0}^{\infty} (1-\theta)\theta^i \beta^{i+1} \Lambda_{t,t+1+i} \left[(R_{k,t+1+i} - R_{t+1+i}) Q_{t+i} S_{j,t+i} + R_{t+1+i} N_{j,t+i} \right] \quad (8)
\end{aligned}
$$

C.3 Capital Producing Firms

The preference equation for capital producing firms is given by

$$
\max E_t \sum_{\tau=t}^{\infty} \beta^{T-\tau} \Lambda_{t,\tau} \left[(Q_\tau - 1) I_{n,\tau} - f\left(\frac{I_{n,\tau} + I_{ss}}{I_{n,\tau-1} + I_{ss}} \right) (I_{n,\tau} + I_{ss}) \right] \quad (26)
$$

with

$$
I_{n,t} \equiv I_t - \delta(U_t) \xi_t K_t
$$

The value function for capital producing firms to optimise with respect to net capital created is therefore:

$$
V(I_{n,t-1}) = \max_{I_{n,t}} (Q_t - 1) I_{n,t} - f\left(\frac{I_{n,t} + I_{ss}}{I_{n,t-1} + I_{ss}} \right) (I_{n,t} + I_{ss}) + \beta E_t \Lambda_{t,t+1} V(I_{n,t})
$$

The first order condition for net investment gives the "Q" relation as shown:

$$\frac{\delta V(I_{n,t-1})}{\delta I_{n,t}} = 0$$

$$\begin{aligned}
0 &= Q_t - 1 - \left[f'\Big(\frac{I_{n,t}+I_{ss}}{I_{n,t-1}+I_{ss}}\Big)(\frac{1}{I_{n,t-1}+I_{ss}})(I_{n,t}+I_{ss}) + f\Big(\frac{I_{n,t}+I_{ss}}{I_{n,t-1}+I_{ss}}\Big)\right] + \beta E_t \Lambda_{t,t+1}\frac{\delta V(I_{n,t})}{\delta I_{n,t}} \\
0 &= Q_t - 1 - f'\Big(\frac{I_{n,t}+I_{ss}}{I_{n,t-1}+I_{ss}}\Big)\Big(\frac{I_{n,t}+I_{ss}}{I_{n,t-1}+I_{ss}}\Big) - f\Big(\frac{I_{n,t}+I_{ss}}{I_{n,t-1}+I_{ss}}\Big) \\
&\quad + \beta E_t \Lambda_{t,t+1}\left[-f'\Big(\frac{I_{n,t+1}+I_{ss}}{I_{n,t}+I_{ss}}\Big)\Big(-\frac{I_{n,t+1}+I_{ss}}{(I_{n,t}+I_{ss})^2}\Big)(I_{n,t+1}+I_{ss})\right] \\
\therefore Q_t &= 1 + f\Big(\frac{I_{n,t}+I_{ss}}{I_{n,t-1}+I_{ss}}\Big) + \Big(\frac{I_{n,t}+I_{ss}}{I_{n,t-1}+I_{ss}}\Big)\cdot f'\Big(\frac{I_{n,t}+I_{ss}}{I_{n,t-1}+I_{ss}}\Big) \\
&\quad - \beta E_t \Lambda_{t,t+1}\Big(\frac{I_{n,t+1}+I_{ss}}{I_{n,t}+I_{ss}}\Big)^2 \cdot f'\Big(\frac{I_{n,t+1}+I_{ss}}{I_{n,t}+I_{ss}}\Big)
\end{aligned}$$

which is, for simplicity sake, written in the paper as:

$$Q_t = 1 + f_t\Big(\cdot\Big) + \Big(\frac{I_{n,t}+I_{ss}}{I_{n,t-1}+I_{ss}}\Big) f'_t\Big(\cdot\Big) - \beta E_t \Lambda_{t,t+1}\Big(\frac{I_{n,t+1}+I_{ss}}{I_{n,t}+I_{ss}}\Big)^2 f'_{t+1}(\cdot) \tag{27}$$

C.4 Retail Firms

Differentiated retailers each face the optimisation of a pricing problem, where the goal is to each choose their optimal reset price, $P^*_{f,t}$, resulting in the industry finding an optimal price as defined by:

$$P^*_t = \left[\int_0^1 (P^*_{f,t})^{1-\varepsilon} df\right]^{1/1-\varepsilon}$$

Thus, the first order condition for the industry to satisfy is given by:

$$0 = \sum_{i=0}^{\infty} \gamma^i \beta^i \Lambda_{t,t+i}\left[\frac{P^*_t}{P_{t+i}}\prod_{k=1}^{i}(1+\pi_{t+k-1})^{\gamma_p} - \mu P_{m,t+i}\right] Y_{f,t+i} \tag{32}$$

D Dynare Code

```
var
Y Ym K Keff L I C G Q varrho Lambda Rk R N Ne Nn nu eta phi z x Pm w
VMPK U X D F Z i prem delta In Welf infl inflstar a ksi g;

varexo
e_a e_ksi e_g e_Ne e_i;

parameters
betta sig hh varphi zetta theta alfa G_over_Y eta_i epsilon gam gam_P
kappa_pi kappa_y rho_i rho_ksi sigma_ksi rho_a sigma_a rho_g sigma_g
sigma_Ne sigma_i rho_shock_psi sigma_psi kappa tau omega lambda chi b
delta_c G_ss I_ss Pmf L_mom RkmR_mom phi_mom  L0 K0 chi0 lambda0
omega0 deltai Y_ss;

//              Exogenous chosen parameters
betta       =   0.99;       //Discount rate
sig         =   1;          //Intertemporal elasticity of substitution
hh          =   0.815;      //Habit formation parameters
varphi      =   0.276;      //Inverse Frisch elasticity of labor supply
zetta       =   7.2;        //Elasticity of marginal dep. of utilization rate
alfa        =   0.33;       //Capital share
deltai      =   0.025;      //Depreciation rate
G_over_Y    =   0.2;        //Government expenditures over GDP
eta_i       =   1.728;      //Elasticity of investment adjusment cost
theta       =   0.97155955; //The survival probability

//              Retail firms
epsilon     =   4.167;      //Elasticity of substitution between goods
gam         =   0.779;      //Calvo parameter
gam_P       =   0.241;      //Price indexation parameter
Pmf         =   log((epsilon-1)/epsilon);

//              Monetary Policy parameters
rho_i       =   0.8;        //Interest rate smoothing parameter
kappa_pi    =   1.5;        //Inflation coefficient
kappa_y     =   -0.5/4;     //Output gap coefficient
```

```
//              Shocks
sigma_ksi   =   0.05;       //Size of the capital quality shock
rho_ksi     =   0.66;       //Persistence of the capital quality shock
sigma_a     =   0.01;       //Size of the TFP shock
rho_a       =   0.95;       //Persistence of the TFP shock
sigma_g     =   0.01;       //Size of the government expenditure shock
rho_g       =   0.95;       //Persistence of the government expenditure shock
sigma_Ne    =   0.01;       //Wealth shock
sigma_i     =   0.01;       //Monetary policy shock
rho_shock_psi=  0.66;       //Persistence of the CP shock
sigma_psi   =   0.072;      //Size of the CP shock

kappa       =   10.0;
tau         =   0.001;
deltai      =   0.025;

//              Targeted moments
L_mom       =   1/3;        //Steady state labor supply
RkmR_mom    =   0.01/4;     //Steady state premium
phi_mom     =   4;          //Steady state leverage

//              Credit policy parameters
kappa       =   10;         //Credit policy coefficient
tau         =   0.001;      //Costs of credit policy

//              Starting values for some steady state values
L0          =   L_mom;
K0          =   9.5;

//              Endogeous determined parameters
lambda0     =   0.3815;     //Starting value divertable fraction
omega0      =   0.002;      //Starting value: proportional starting up funds
chi0        =   3.4;        //Starting value for the labor utility weight

model;

//              Households
exp(varrho)
     = (exp(C)-hh*exp(C(-1)))^(-sig)-betta*hh*(exp(C(+1))-hh*exp(C))^(-sig);
```

```
betta*exp(R)*exp(Lambda(+1))   =    1;
exp(Lambda) =    exp(varrho)/exp(varrho(-1));
chi*exp(L)^varphi      =    exp(varrho)*exp(Pm)*(1-alfa)*exp(Y)/exp(L);

//              Financial Intermediaries
exp(nu)       =    (1-theta)*betta*exp(Lambda(+1))*(exp(Rk(+1))-exp(R))
                   +betta*exp(Lambda(+1))*theta*exp(x(+1))*exp(nu(+1));
exp(eta)      =    (1-theta)+betta*exp(Lambda(+1))*theta*exp(z(+1))*exp(eta(+1));
exp(phi)      =    exp(eta)/(lambda-exp(nu  ));
exp(z)        =    (exp(Rk  )-exp(R(-1)))*exp(phi(-1))+exp(R(-1));
exp(x)        =    (exp(phi)/exp(phi(-1)))*exp(z);

//              Aggregate capital, net worth
exp(Q)*exp(K)=   exp(phi)*exp(N);
exp(N)        =    exp(Ne)+exp(Nn);
exp(Ne)       =    theta*exp(z)*exp(N(-1))*exp(-e_Ne);
exp(Nn)       =    omega*exp(Q)*exp(ksi)*exp(K(-1));

//              Final goods producer
exp(Rk  )     =    (exp(Pm  )*alfa*exp(Ym  )/exp(K  (-1))
                   +exp(ksi)*(exp(Q  )-exp(delta  )))/exp(Q  (-1));
exp(Ym)       =    exp(a)*(exp(ksi)*exp(U)*exp(K(-1)))^alfa*exp(L)^(1-alfa);

//              Capital Goods Producer
exp(Q)        =    1+eta_i/2*((In+I_ss)/(In(-1)+I_ss)-1)^2+eta_i*((In+I_ss)
                   /(In(-1)+I_ss)-1)*(In+I_ss)/(In(-1)+I_ss)
                   -betta*exp(Lambda(+1))*eta_i*((In(+1)+I_ss)
                   /(In+I_ss)-1)*((In(+1)+I_ss)/(In+I_ss))^2;
exp(delta)    =    delta_c+b/(1+zetta)*exp(U)^(1+zetta);
exp(Pm)*alfa*exp(Ym)/exp(U)
              =    b*exp(U)^zetta*exp(ksi)*exp(K(-1));
In            =    exp(I)-exp(delta)*exp(ksi)*exp(K(-1));
exp(K)        =    exp(ksi)*exp(K(-1))+In;
exp(G)        =    G_ss*exp(g);

//              Equilibrium
exp(Y)        =    exp(C)+exp(G)+exp(I)+eta_i/2*((In+I_ss)
                   /(In(-1)+I_ss)-1)^2*(In+I_ss);
exp(Ym)       =    exp(Y)*exp(D);
```

```
exp(D)          =    gam*exp(D(-1))*exp(infl(-1))^(-gam_P*epsilon)*exp(infl)
                     ^epsilon+(1-gam)*((1-gam*exp(infl(-1))^(gam_P*(1-gam))
                     *exp(infl)^(gam-1))/(1-gam))^(-epsilon/(1-gam));
exp(X)          =    1/exp(Pm);
exp(F)          =    exp(Y)*exp(Pm)+betta*gam*exp(Lambda(+1))*exp(infl(+1))
                     ^epsilon*(exp(infl))^(-epsilon*gam_P)*exp(F(+1));
exp(Z)          =    exp(Y)+betta*gam*exp(Lambda(+1))*exp(infl(+1))
                     ^(epsilon-1)*exp(infl)^(gam_P*(1-epsilon))*exp(Z(+1));
exp(inflstar)=   epsilon/(epsilon-1)*exp(F)/exp(Z)*exp(infl);
(exp(infl))^(1-epsilon)    =      gam*exp(infl(-1))^(gam_P*(1-epsilon))
                     +(1-gam)*(exp(inflstar))^(1-epsilon);
exp(i)          =    exp(R)*exp(infl(+1));
exp(i)          =    exp(i(-1))^rho_i*((1/betta)*exp(infl)^kappa_pi*(exp(X)
                     /(epsilon/(epsilon-1)))^(kappa_y))^(1-rho_i)*exp(e_i);

//                   Extra Variables
exp(Keff)       =    exp(ksi)*exp(K(-1));
exp(w)          =    exp(Pm)*(1-alfa)*exp(Y)/exp(L);
exp(VMPK)       =    exp(Pm)*alfa*exp(Y)/(exp(ksi)*exp(K(-1)));
Welf            =    log(exp(C)-hh*exp(C(-1)))-chi*exp(L)^(1+varphi)
                     /(1+varphi)+betta*Welf(+1);
exp(prem)       =    exp(Rk(+1))/exp(R);

//                   Shocks
a               =    rho_a*a(-1)-e_a;
ksi             =    rho_ksi*ksi(-1)-e_ksi;
g               =    rho_g*g(-1)-e_g;

end;
check;
shocks;
var e_a         =    1;
var e_ksi       =    sigma_ksi^2;
var e_g         =    sigma_g^2;
var e_Ne        =    1;
var e_i         =    0.25^2;
end;

stoch_simul(order=1, irf=40, nograph, noprint);
```